



Calorimeter Basics

- a short tutorial

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1. Overview of Thermal Detectors

2. Basics of Near-Equilibrium Detectors

- Classical Bolometers and Calorimeters
- Resistive thermometers

Ionization Detectors

- Measure only part of energy that goes into ionization (~1/3)
- Statistical fluctuations in branching ratio limit resolution (~115 eV FWHM at 6 keV for silicon)
- Require good electron transport properties:
 - ⇒ severe material restrictions
 - ⇒ low tolerance of impurities



Low Temperature Detectors

1. Superconducting Tunnel Junctions (STJ's):

- Analog of semiconductor ionization detector
- $1000 \times$ smaller gap $\Rightarrow \sim 30 \times$ better resolution
- More materials — still have transport problems

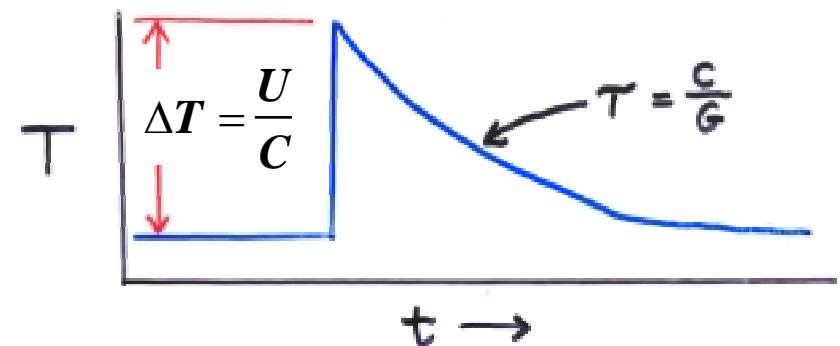
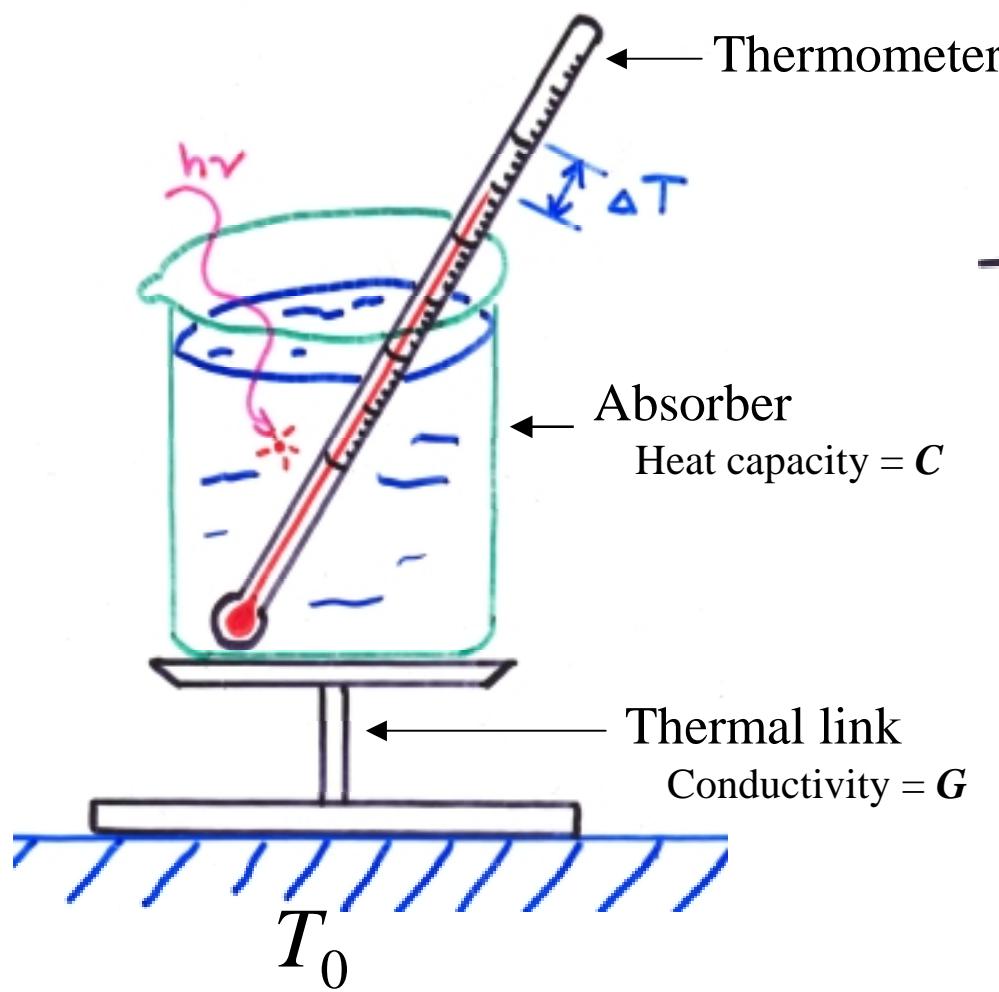
2. Non-equilibrium Phonon Detectors

- Wide selection of materials
- Sensitivity to non-ionizing events

3. Near-equilibrium Thermal Detectors (Bolometers and Calorimeters):

- No branching \Rightarrow no intrinsic resolution limit
- Few material restrictions
- High tolerance for impurities

Thermal Calorimetry





Thermometer Technologies

1. Thermistors $R(T)$

- ↓ Johnson noise
- ↓ Readout power
- ↑ High Sensitivity

2. Capacitive $C(T)$

3. Inductive $L(T)$

- kinetic inductance:

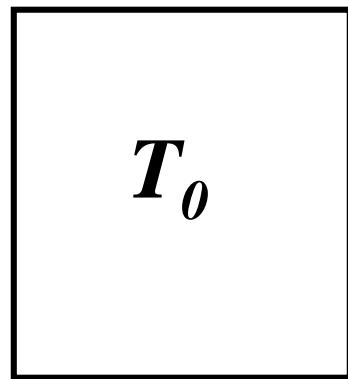
4. Magnetization $M(T)$

5. Pyro-Electric $V(T)$

No bias power!

6. ??

“Fundamental” Noise Sources and Resolution Limits



$$\langle \mathbf{U} \rangle \pm \Delta \mathbf{U}$$

$$\langle \Delta \mathbf{U}^2 \rangle = k_B T^2 C$$

(note independent of G)

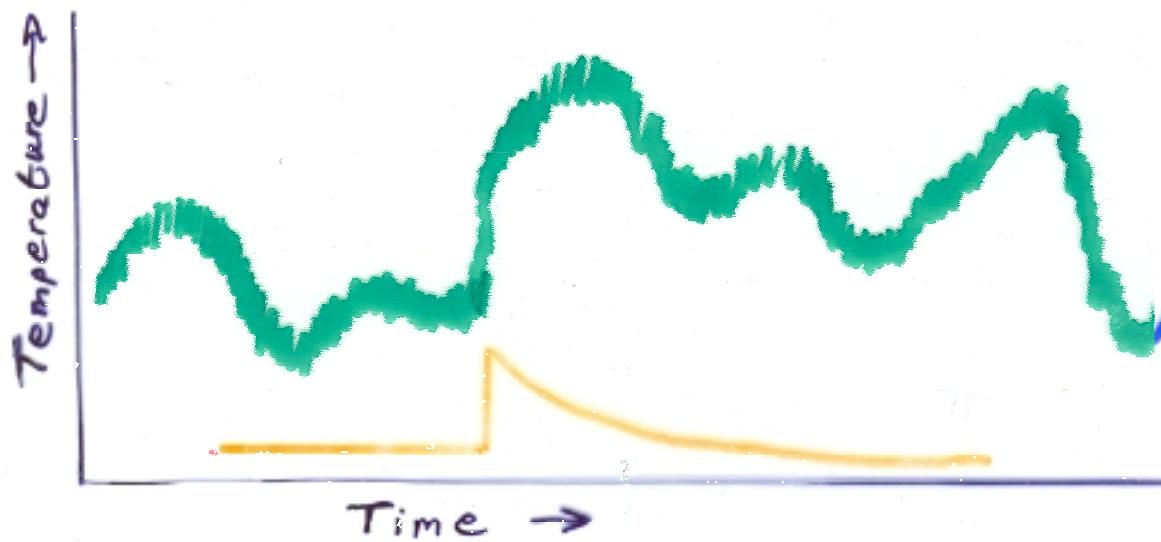
“Thermodynamic fluctuation noise” (TFN)

$$C \propto T^\gamma, \quad \gamma = 1 - 3$$

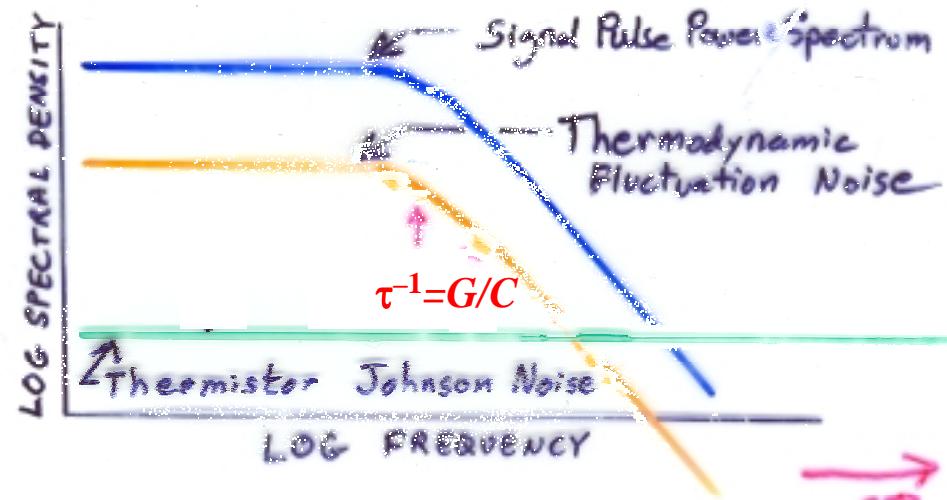
\Rightarrow big advantage to low T

~Poisson fluctuations:

$$N \approx \frac{CT}{k_B T} \quad \Delta U_{rms} = \sqrt{N} \cdot (k_B T) = \sqrt{k_B T^2 C}$$

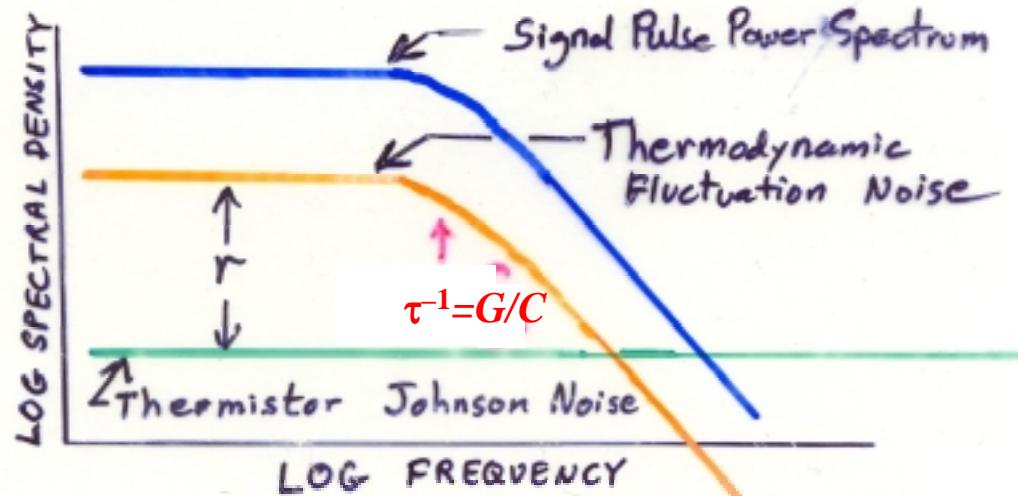


TFN is ***not*** in itself a limit to energy resolution



For stationary noise:

Each frequency bin gives an independent measurement of the signal amplitude.



$$r \approx \alpha \left(\frac{\Delta T}{T_0} \right)^{\frac{1}{2}}$$

$$dV = IdR, \quad dR = \frac{dR}{dT} dT, \quad \alpha \equiv \frac{d \log(R)}{d \log(T)}$$

ΔT has optimum value:
 too small \Rightarrow Johnson noise dominates
 too big \Rightarrow temperature, TFN, and C go up
 $(\Delta T/T)_{\text{OPTIMUM}} \approx 11\% - 18\%$ (mostly depending on γ)

α : Big is good! $f_{\max} \propto r \propto \alpha$, $\Delta U \propto f^{-\frac{1}{2}}$, $\therefore \Delta U \propto \alpha^{-\frac{1}{2}}$



$$\Delta U = \xi \sqrt{k_B T_0^2 C_0}$$

$$t \equiv T / T_0$$

$$C = C_0 t^\gamma$$

$$G = G_0 t^\beta$$

- “0” subscripts are values at *heatsink* temperature
 - Must account for increase in temperature due to bias power
 - Temperature gradient across link modifies fluctuation current

$$\langle \Delta U_{TFN}^2 \rangle = k_B T_{eff}^2 C, \quad T_0 < T_{eff} < T$$

$$\frac{T_{eff}}{T_0} = f(t, \beta, \lambda/L)$$

Useful extreme cases: 1. Diffusive ($\lambda \ll L$)
(beams or membrane)

2. Radiative ($\lambda \gg L$) (hot electron)

$$\xi = \mathbf{F}(\alpha, t, \beta, \gamma, \lambda)$$

- Adjust t to minimize ξ ($t = 1.11\text{--}1.18$)
 - Strong dependence on α , quite weak on others
 - for $\alpha > 6$, $\xi \approx 2\sqrt{6/\alpha}$

Semiconductors: $\alpha \sim 6 - 8$; TES: $\alpha \sim 20 - 1000$

Still no G -dependence
(might have to make it
slow to use high α)

Above Results Assumed Optimal Filtering:

S_i = measured signal in i^{th} frequency bin is an independent estimator of U .

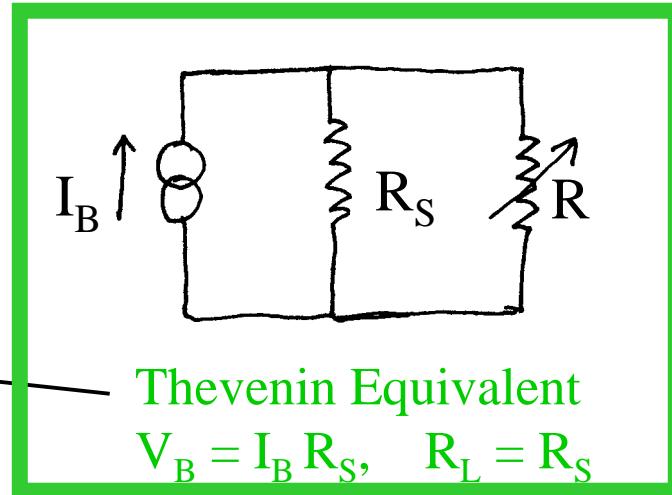
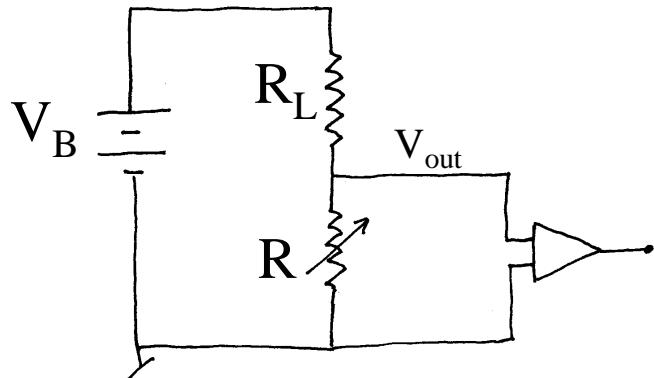
$U_{\text{ESTIMATE}} = \sum_{i=1}^{\infty} w_i s_i$, where w_i are the filter weights. Noise is independent in each frequency bin, therefore the expected rms noise is: $\sqrt{\sum (w_i n_i)^2}$.

Want to maximize: $\frac{U_{\text{EST}}}{\text{noise}} = \frac{\sum w_i s_i}{\sqrt{\sum (w_i n_i)^2}}$, so differentiate w.r.t. w_i

General result for linear detector with stationary noise:

$$\Rightarrow w_i \propto \frac{s_i^*}{n_i^2} \quad \left\langle \Delta U^2 \right\rangle = \frac{1}{4 \int_0^\infty \frac{s_i^*(f)s_i(f)}{n_i^2(f)} df} = \frac{1}{\int_0^\infty \frac{4df}{\text{NEP}^2(f)}}$$

Electrothermal Feedback



$P_{\text{ext}} \uparrow \Rightarrow T \uparrow \Rightarrow R \downarrow$ (for $\alpha < 0$), $P_{\text{bias}} \uparrow$ or \downarrow depending on R_L/R_{det}

$P_{\text{bias}} \uparrow \Rightarrow \text{positive}$ feedback (adds to effect of P_{ext})

$P_{\text{bias}} \downarrow \Rightarrow \text{negative}$ feedback (partly cancels effect of P_{ext})

Negative feedback: $\alpha < 0$ and $R_L > R$, or $\alpha > 0$ and $R_L < R$

Zero feedback: $R_L = R$

SUMMARY

$$L \equiv \frac{\alpha P}{G T}$$

$$K_L = \frac{R_L}{R_L + R}$$

$$b \equiv \frac{R_L - R}{R_L + R} = (2K_L - 1)$$

$$K_F = \left(\frac{1}{1 - bL} \right) \frac{1 + i\omega\tau}{1 + i\omega \frac{\tau}{(1 - bL)}}$$

$$NEP^2 = NEP_{TFN}^2 + NEP_{Johnson}^2 \quad (+NEP_{R_L}^2 + NEP_{IFN}^2 + NEP_{ampl.}^2)$$

$$S(\omega) = \frac{L}{I} \left(\frac{1}{1 + i\omega\tau} \right) K_F K_L$$

$$NEP(\omega) \equiv \frac{e_{n-out}(\omega)}{S(\omega)}$$

$$e_{n-TFN} = \sqrt{4k_B T^2 G} \cdot \frac{L}{I} \left(\frac{1}{1 + i\omega\tau} \right) K_F K_L$$

$$e_{n-Johnson} = \sqrt{4k_B T R} \cdot K_F K_L$$

$$e_{n-R_L} = \sqrt{4k_B T / R_L} \cdot R \left(1 - \frac{L}{1 + i\omega\tau} \right) K_F K_L$$

$$e_{n-amplifier} = e_{n-amp}$$

K_F and K_L cancel in all NEP's except amplifier noise



TIPS

- You can consider the effects of most things on resolution with feedback “turned off” to make spectra simpler.
- Electrothermal feedback makes the spectra more complicated, but most lines move down together.

$$\bullet \text{Output of optimal filter} = \frac{s^*(f)}{n^2(f)} s(f) = \left| \frac{s(f)}{n(f)} \right|^2 = \left| \frac{s}{n}(f) \right|^2$$

That is, it depends only on the S/N ratio as a function of frequency. This is unchanged by feedback, so:

- Turn ETF on: no change in pulse shape out of filter.
- Increase α : pulses out of filter get faster!
- Many practical advantages to large negative ETF:
 - Detector cools faster, so due to nonlinear effects can have better resolution at high count rates.
 - Necessary for practical operation at very high α

What about Current Output?

“Flat-circuit Theorem”

$$V \Leftrightarrow I$$

$$I \Leftrightarrow V$$

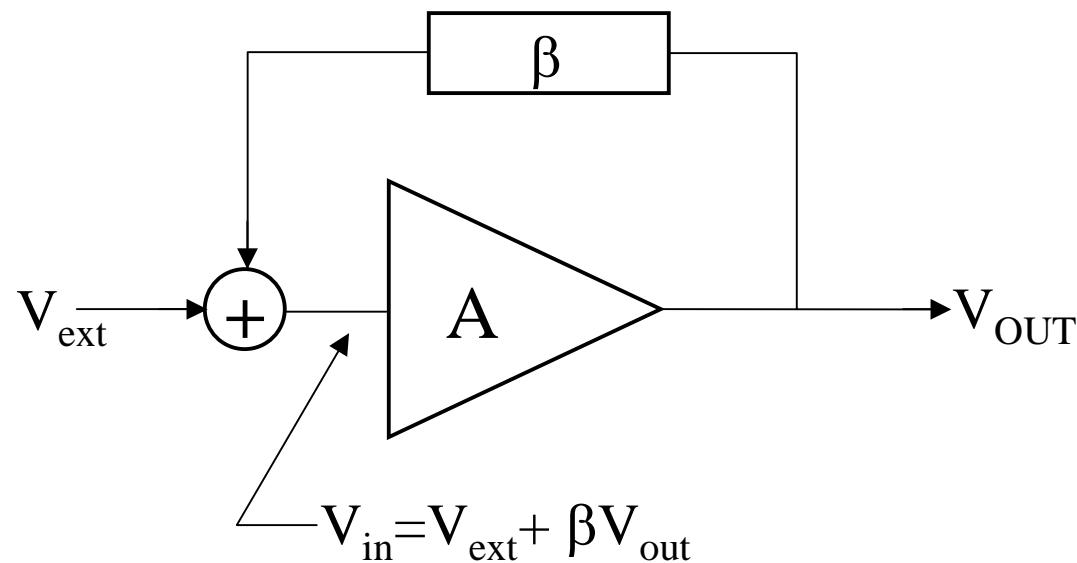
$$R \Leftrightarrow 1/R$$

$$\alpha \Leftrightarrow -\alpha$$

$$K_L \Leftrightarrow 1-K_L$$

$$b \Leftrightarrow -b$$

$$K_F \Leftrightarrow K_F$$



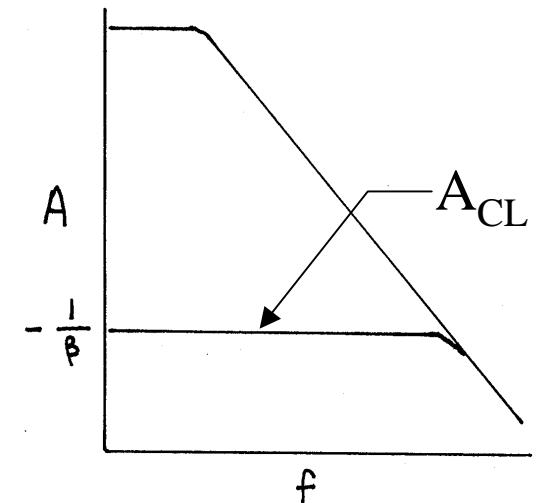
“Open Loop Gain” $\equiv V_{out}/V_{in} = A$

“Loop Gain” $= \beta A$

“Closed Loop Gain” $\equiv V_{out}/V_{ext} = A/(1-\beta A) = A_{CL}$

For $\beta A \ll -1$, $\frac{V_{out}}{V_{ext}} \Rightarrow -\frac{1}{\beta}$, (indep. Of A) = Gain Stabilization

For noise sources in A, β , +, e_{n-out}/A_{CL} = “input-referred noise” = constant independent of β .



$$S(\text{responsivity}) \equiv \frac{dV_{out}}{dP_{in}} = \frac{dT}{dP_{in}} \bullet \frac{dR}{dT} \bullet \frac{dV_{out}}{dR}$$

(No feedback: $=A$)

$$\Delta T = \frac{\Delta P_{in}}{G} \Rightarrow \frac{dT}{dP_{in}} = \frac{1}{G}$$

$$\frac{dR}{dT} = \frac{R}{T} \frac{d \log R}{d \log T} = \frac{R}{T} \alpha$$

$$V_{out} = \frac{R}{R + R_L} V_B \Rightarrow \frac{dV_{out}}{dR} = \frac{R_L}{(R + R_L)^2} V_B = \frac{R_L}{R + R_L} I \equiv K_L I$$

$$S(0) = \frac{1}{G} \bullet \frac{R}{T} \alpha \bullet I K_L \bullet \frac{I}{I} = \frac{1}{I G T} K_L \equiv \frac{1}{I} L K_L$$

Electrical \Leftrightarrow Thermal

$$V \Leftrightarrow T$$

$$R \Leftrightarrow 1/G$$

$$I \Leftrightarrow P$$

$$Q \Leftrightarrow U$$

$$C \Leftrightarrow C$$

$$V = IR \quad \Leftrightarrow \quad T = \frac{P}{G}$$

$$\dot{W} = \frac{I}{C} \quad \Leftrightarrow \quad \dot{Y} = \frac{P}{C}$$



Now do $S(\omega)$ — still no feedback:

$$I = \frac{V}{R} + C\dot{V}$$

assume $V = V_0 e^{i\omega t}$

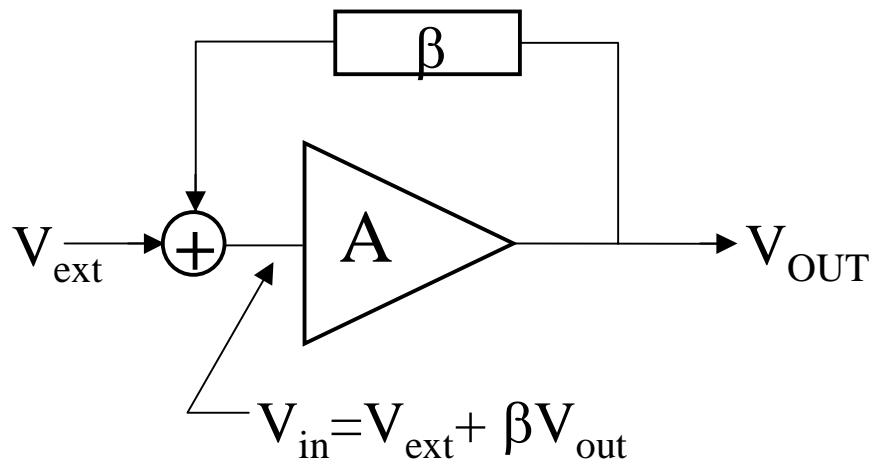
$$\dot{V} = i\omega V$$

$$I = V \left(\frac{1}{R} + i\omega C \right)$$

$$\frac{dV}{dI} = R \frac{1}{1 + i\omega RC}$$

$$\frac{dT}{dP_{in}} = \frac{1}{G} \frac{1}{1 + i\omega \frac{C}{G}} \equiv \frac{1}{G} \boxed{\frac{1}{1 + i\omega \tau}}$$

Now put in feedback:



$$\beta = \frac{dP_{Bias}}{dV_{OUT}}, \quad \because P_{Bias} = \frac{V_{OUT}^2}{R}$$



$$\begin{aligned} \therefore \frac{dP_{Bias}}{dV_{OUT}} &= \frac{2RV_{OUT} - V_{OUT}^2 \frac{dR}{dV_{OUT}}}{R^2} \\ &= I \left(2 - \frac{1}{K_L} \right) \equiv \beta \end{aligned}$$

$$\therefore \text{"Loop Gain"} \quad \beta A = I \left(2 - \frac{1}{K_L} \right) \frac{L}{I} \cdot K_L \left(\frac{1}{1+i\omega\tau} \right) = bL \left(\frac{1}{1+i\omega\tau} \right)$$

$$\text{where } b \equiv (2K_L - 1) = \frac{R_L - R}{R_L + R}$$

$$A_{CL} = S(\omega) = \frac{A}{1 - \beta A} \equiv AK_F,$$

K_F = "feedback factor"

$$K_F = \frac{1}{1 - \beta A} = \left(\frac{1}{1 - bL} \right) \frac{1 + i\omega\tau}{1 + i\omega \frac{\tau}{(1 - bL)}}$$

$$\begin{aligned} S(\omega) &= AK_F = \frac{L}{I} \left(\frac{1}{1 + i\omega\tau} \right) K_F K_L \\ &= \frac{L}{I(1 - bL)} K_L \left(\frac{1}{1 + i\omega\tau_e} \right), \quad \tau_e \equiv \frac{\tau}{1 - bL} \end{aligned}$$



External Electrothermal Feedback

$$\beta A = bL \quad b = \frac{R_L - R}{R_L + R}$$

Easiest to look at it as feedback signal effectively makes R_L negative. Value can be controlled by adjusting β_2 .

Note that amplifier noise is inside this loop, so it is reduced by feedback along with signal and other noise sources.